(NASA-TM-85039) THE NEED FOR AN INTERMEDIATE MASS SCALE IN GUTS (NASA) 16 p HC A02/MF A01 CSCL 03B
N83-29151
Unclas
G3/90 03978

NASA

Technical Memorandum **85039**

The Need for an Intermediate Mass Scale in GUTs

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May 1983

National Aeronautics and
Space Administration

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THE NEED FOR AN INTERMEDIATE MASS SCALE IN GUTS*

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Abstract

The minimal SU(5) GUT model fails to resolve the strong CP problem, suffers from the cosmological monopole problem, sheds no light on the nature of the "dark" mass in the universe, and predicts an unacceptably low value for the baryon asymmetry. All these problems can be overcome in one fell swoop in suitable grand unified axion models with an intermediate mass scale of about 10^{11}–10^{12} GeV. An example based on the gauge group SO(10) is presented. Among other things, it predicts that the axions comprise the "dark" mass in the universe, and that there exists a galactic monopole flux of 10^{-8} - 10^{-7} cm^{-2} yr^{-1}. Other topics that are briefly discussed include proton decay, family symmetry, neutrino masses and the gauge hierarchy problem.

Despite the remarkable successes enjoyed by the standard SU(3)xSU(2)xU(1) gauge model in describing electroweak and strong interaction phenomenon at present energies, there are good reasons to suspect that the model is only a part of a more complete theory. Let me point out some of them:

*Invited talk presented at the Europhysics Study Conference on Electroweak Effects at High Energies, Erice, Feb. 1983.

i) The model involves three independent gauge couplings g_3, g_2 and g_1, associated with SU(3), SU(2) and U(1) respectively. If the couplings could be related, $\sin^2\theta_w$ can be predicted.

ii) Charge quantization (in units of e/3, where e is the electron charge) is put in by hand.

iii) Model allows, in principle, leptons with fractional charges or color triplet fermions with integer charges. Such particles are not found.

iv) The model does not explain why $\bar{\theta}_{QCD} < 10^{-9}$.

v) The origin of fermion masses, mixing angles etc. is left unexplained.

vi) The standard model fails to shed light on several important problems in cosmology, such as the origin of baryon asymmetry in the universe, the nature of the dark mass in the universe, etc.

A promising approach for resolving at least some of these questions is offered by grand unified theories (or GUTS, for short). The basic idea is to embed the standard model in a larger gauge group.[1] The simplest GUT model is based on SU(5) which is a rank four group.[2] This model nicely takes care of points (i), (ii) and (iii) listed above. It makes some other interesting predictions such as the occurence of baryon number violating processes and superheavy ($\sim 10^{16}$ GeV) magnetic monopoles. But there are problems with the minimal model. Below I list a few of them, some taken from particle physics and some others from cosmology.

Particle Physics	SU(5)	Suggested Cures Include
Strong CP problem ($\bar{\Theta}_{QCD} < 10^{-9}$)	U(1) global chiral symmetry; but does not work(see later)	GUTS with an intermediate mass scale; Supersymmetry
Gauge Hierarchy problem ($m_W/m_X \sim 10^{-13}$)	Fine tune to each order in the perturbation expansion	Supersymmetry
Fermion masses; Mixing Angles	$m_b \simeq 3m_\tau$; Some bad relations or no predictions	Family Symmetry; Kaluza-Klein approach (e.g. N=8 Supergravity)
Higgs sector	Largely arbitrary	Dynamical symmetry breaking; Kaluza-Klein

Cosmology	SU(5)	Suggested cures include
n_b/n_γ (baryon asymmetry)	Orders of magnitude too low	Extended higgs sector; Larger GUTS
Dark mass (Missing mass problem)	No non-baryonic candidate	Larger GUTS with massive neutrinos; axions, photino, gravitino, higgsino.
Primordial magnetic monopoles	Too many	Inflation (but cannot be implemented in SU(5)); Larger GUTS with an intermediate mass scale;
$\delta\rho/\rho$ (density perturbations in the very early universe)	Assuming inflation get $\delta\rho/\rho >> 10^{-4}$ which is unacceptable	Extended structures (e.g. strings) from larger GUTS; Gravity
Cosmological constant $\Lambda \simeq o$.	Fine tune	Kaluza-Klein

My intention here is to argue that at least some of these problems (in particular the strong CP and the cosmological monopole problems) can be nicely resolved by introducing an intermediate scale of about 10^{11}–10^{12} GeV in grand unification theories. Clearly, this entails going beyond SU(5).

Strong CP problem and the Peccei-Quinn Mechanism

The strong CP problem arises because non-perturbative QCD effects force one to add to the standard SU(3)xSU(2)xU(1) Lagrangian an extra term ΔL given by[3]

$$\Delta L = \bar{\theta} \frac{g^2}{32\pi^2} F^a_{\mu\nu} \tilde{F}^a_{\mu\nu}$$

where $\bar{\theta} = \theta + \arg \det M$. Here θ is the angle that characterizes the QCD vacuum and M is the quark mass matrix in the weak eigenstate basis. Clearly, the extra term ΔL violates CP and its presence induces an electric dipole moment of the neutron, current experimental upper limits on which require that $\bar{\theta} < 10^{-9}$. How $\bar{\theta}$ happens to have such a small value is not explained by the stardard SU(3)xSU(2)xU(1) gauge model and is referred to as the strong CP problem.

The small value of $\bar{\theta}$ is most naturally understood, as we shall see, in those models that possess a spontaneously broken global chiral U(1) (Peccei-Quinn) symmetry.[4] Briefly, the Peccei-Quinn mechanism works as follows. Under U(1) transformations not only the fermion fields but also the scalar fields transform suitably, such that the classical Lagrangian is U(1) invariant. The U(1) symmetry, however, is explicitly broken by QCD instanton effects, with the consequence that $\bar{\theta}$ also transforms under U(1) rotations, i.e., $\bar{\theta}$ becomes one of the dynamical variables. The potential energy contains a term proportional to

$$\Lambda^4 (1-\cos\bar{\theta}), \quad \Lambda \simeq 100\text{MeV is the QCD scale,}$$

which is minimized for $\langle\bar{\theta}\rangle = o$. The strong CP problem is no more!

The spontaneous breaking of the U(1) symmetry (broken only by QCD instanton effects) leads to the appearance of a pseudo-Goldstone boson known as the axion.[5] The axion has a mass $m_a \sim f_\pi m_\pi / f_A$, where f_A is the dominant U(1) breaking scale. The important question now is: What is f_A?

The axion is consistent with all known laboratory constraints for $f_A \gtrsim 1$ TeV. A more stringent constraint on f_A comes from astrophysical considerations. In order that the power radiated in axions by the helium core of a red giant be not too excessive, one requires that $f_A \gtrsim 10^8$ GeV.[6] So we conclude that the axion must be

light ($\lesssim 10^{-1}$ eV), and also weakly coupled (its couplings to ordinary matter should be suppressed by inverse powers of f_A). Remarkably enough, there is even an upper bound on f_A which comes from cosmology.[7] In order to derive it, we must briefly consider axion production and their subsequent evolution in the early universe.

Primordial Axions

At $T \sim f_A$, the U(1) symmetry is spontaneously broken by $\langle\phi\rangle \sim f_A\, e^{i\bar\theta}$, with $\bar\theta$ taking some value between 0 and 2π. The axion field is defined to be $\phi_A \equiv f_A\bar\theta$. For $\Lambda < T < f_A$, $L(\phi_A) \sim (\partial_\mu \phi_A)^2$ so that the axion field essentially behaves as a free massless scalar field. For $T \lesssim \Lambda$, the QCD instanton effects introduce the term $\Lambda^4(1-\cos\bar\theta)$ and the field ϕ_A starts to perform damped oscillations about $\langle\bar\theta\rangle=0$. These oscillations produce a coherent state of axions at rest. They turn out to be non-relativistic even though they are produced at $T \sim \Lambda \gg m_a$! The energy density in the axion gas decreases as R^{-3}, whereas the radiation energy density falls off as R^{-4}. Imposing the requirement

$$\rho_a \equiv \rho_{axions} \lesssim (1\text{-}10)\,\rho_c \quad (\rho_c \sim 2 \times 10^{-29}\ \mathrm{g\ cm^{-3}}\ \text{is the critical energy density})$$

one obtains the promised upper bound on f_A,[7]

$$f_A \lesssim 10^{11}\text{-}10^{12}\ \mathrm{GeV}$$

Thus, the Peccei-Quinn mechanism can be satisfactorily implemented only if we are prepared to introduce an intermediate mass scale in GUTS. We must go beyond SU(5).

Before discussing other problems from our list, we must consider another constraint on axion models that arises from cosmological considerations. In the effective field theory describing physics at ordinary energies, the Peccei-Quinn U(1) symmetry is realized non-linearly. Under a U(1) transformation $e^{i\psi Q}$,

$$\bar\theta \to \bar\theta - 12\psi$$

provided there are three fermion familes. For $\psi = n\pi/6$, $\bar\theta \to \bar\theta + 2\pi n$ which is an identity transformation. It follows that there is a discrete subgroup of U(1), consisting actually of six distinct elements, which is not broken by QCD effects, but which is spontaneously broken by higgs vacuum expectation values. The spontaneous breaking of the discrete symmetry implies the existence of topologically stable domain walls which are cosmologically unacceptable[8]. Thus, we are confronted with a domain wall problem.

Let us summarize what we have learnt so far. In order to satisfactorily implement the Peccei-Quinn mechanism, we must ensure that

1) the spontaneous breaking scale f_A of U(1) satisfies

$$10^8 \text{ GeV} \lesssim f_A \lesssim 10^{12} \text{ GeV}$$

2) there are no topologically stable domain walls.

Several remarks are now in order:

A) It has already been mentioned that (1) can be taken care of by going to GUTS larger than SU(5), e.g., SO(10).

B) The resolution of (2) necessarily involves the introduction of new fermions that transform under real representations of the gauge group.[9,10] The vacuum structure of the theory can then be made topologically trivial, and the domain wall problem is avoided. In some models the U(1) symmetry prevents the additional fermions from acquiring huge masses through direct coupling to higgs that acquire large vacuum expectation values. Radiatively acquired masses, in two or three loops, then make these fermions relatively light, of order 10^2–10^3 GeV. It is important to look for such fermions in the next generation of high energy machines. A characteristic signature would be their V+A couplings to the W bosons.

C) An elegant resolution of (2) involves embedding the unbroken discrete elements of U(1) in a continuous symmetry which is most naturally identified with a family symmetry[9]. The family symmetry could either be global or local.

D) Suppose it is global and also spontaneously broken.[11] This then implies the existence of goldstone bosons called familons. These objects can be looked for in rare decays such as

$$\mu^- \to e^- + f \text{ (familon)}$$

$$K^+ \to \pi^+ + f$$

One expects

$$\frac{\Gamma(\mu^- \to e^- + f)}{\Gamma(\mu^- \to e^- + \nu_\mu + \bar{\nu}_e)} \sim 2.5 \times 10^{14} \left(\frac{\text{GeV}}{F}\right)^2 \text{ etc.,}$$

where F is the relevant symmetry breaking scale. Typically, $F \sim 10^{11}$–10^{12} GeV.

E) For $f_A \sim 10^{11} - 10^{12}$ GeV, $\rho_a \sim (1-10)\ \rho_c$. A new cosmological scenario where axions provide the dark matter in the universe has recently been constructed. Axion models predict the existence of topologically unstable extended structures called "walls bounded by strings". Fluctuations ($\delta\rho/\rho$) in the axion field energy density produced by these structures may cause the appearance of "axion clumps" with masses $\sim 10^6 M_\odot$.[12] These objects would then form the "building blocks" for a clustering hierarchy theory of galaxy and supercluster formation on length scales up to 10 Mpc and mass $\sim 10^{15} M_\odot$.[12,13] They also provide the seed potential wells needed for galaxy formation.

Thus, in axion models we may not have to postulate an arbitrary spectrum of initial density perburbations $\delta\rho/\rho$. The latter may come naturally and causally from the physics of the U(1) symmetry breaking which produces the axions to begin with. Another problem on our list can therefore been taken care of!

Next let me discuss a specific SO(10) model which satisfactorily implements the Peccei-Quinn mechanism and also possesses other interesting features.

An SO(10) Model

Consider the following breaking of SO(10)[14]

$$SO(10) \xrightarrow[M_X]{} SU(4)_c \times SU(2) \times U(1) \xrightarrow[M_C]{} SU(3) \times SU(2) \times U(1)_y \xrightarrow[M_W]{} SU(3) \times U(1)_{em}$$

The first step in the symmetry breaking can be achieved by a combination of a real $\underline{45}'$ and a real $\underline{54}$, both with PQ charge zero. The second breaking requires $\underline{126}$, $\underline{45}$ and $\underline{16}$ of Higgs fields, with PQ charges 2, 4, and zero respectively. The U(1) symmetry is also broken at this stage. Finally, a $\underline{10}$ plet of Higgs field with PQ charge −2 can achieve the last step in the symmetry breaking. The fermion content of the model is given by

$$\psi_{16}^{(i)}\ (i=1,2,3),\ \psi_{10}^{(\alpha)}\ (\alpha=1,2)$$

where the subscripts denote the dimension of the SO(10) representation to which the various fields belong. The U(1) transformation properties of the fermion fields are:

$$\psi_{16}^{(i)} \to e^{i\theta}\ \psi_{16}^{(i)}$$

$$\psi_{10}^{(\alpha)} \to e^{-2i\theta}\ \psi_{10}^{(\alpha)}$$

and are chosen so that the residual, discrete PQ symmetry coincides with the center Z_4 of SO(10). Had we not included $\psi_{10}^{(\alpha)}$ (α=1,2), the residual PQ symmetry would be Z_{12} which, of course, is too large to be embedded in Z_4.

We next use the one loop renormalization group equations for the various couplings constants to calculate M_x and M_c in terms of $\sin^2\theta_w(M_w)$ and $\alpha_s(M_w)$. We have included the following Higgs contributions. Between M_c and M_x we include the $(\overline{10},1,-1)$ component of $\underline{126}$, the (15,1,0) component of $\underline{45}$ and the (1,2,½) component of $\underline{10}$. Between M_w and M_c we include only the Weinberg-Salam doublet. The fermions in the $\underline{10}$ contribute to the renormalization group equations between M_c and M_x. The results for M_x and M_c are shown in Table I.

Table I

$\sin^2\theta_w(M_w)$	$\alpha_s^{-1}(M_w)$	M_c(GeV)	M_x(GeV)
0.22	7.5	3.5×10^{11}	2×10^{15}
0.22	8.0	6.2×10^{11}	1.3×10^{15}
0.22	9.0	1.9×10^{12}	5.7×10^{14}
0.23	7.5	3.7×10^{9}	2.6×10^{15}
0.23	8.0	6.6×10^{9}	1.7×10^{15}
0.23	9.0	2.0×10^{10}	7.2×10^{14}

Table I: M_c and M_x as functions of $\sin^2\theta_w(M_w)$ and $\alpha_s(M_w)$ with $SU(4)_c \times SU(2) \times U(1)$ as the intermediate symmetry group. The U(1) Peccei-Quinn symmetry is broken at scale M_c.

For the sake of definiteness, from now on we concentrate on the first possibility in Table I i.e. $M_c \sim 3.5\times10^{11}$ GeV and $M_c \sim 2\times10^{15}$ GeV, which corresponds to $\sin^2\theta_w = 0.22$ and $\alpha_s(M_w) = 0.13$. It follows from our previous considerations that $\rho_{axions} \sim \rho_c$.

Let us next see how the SO(10) axion model takes care of some of the other problems on our list. Consider first the cosmological monopole problem in the context of SU(5).

For T > 10^{15} GeV, the expectation value of the Higgs 24-plet ϕ is zero so that the SU(5) gauge symmetry is restored. As the

universe expands and cools below $T \sim 10^{15}$ GeV, $\langle\phi\rangle$ starts to acquire a non zero vacuum expectation value. Assuming a weakly first order phase transition, the Higgs field is rapidly quenched. Monopoles are produced during this phase transition as topological knots (Kibble mechanism). One estimates the initial relative monopole number density to be

$$r_{in} \equiv (n/T^3)_{in} \gtrsim 10^{-10}$$

where n denotes the number density of the monopoles (and antimonopoles). Subsequent monopole - antimonopole annihilation cannot reduce this much below[15]

$$r_f \sim 10^{-10}$$

Thus, one expects to find roughly one monopole per baryon. Needless to say, this is cosmologically unacceptable and is referred to as the cosmological monopole problem.

Many attempts have been made to overcome the problem in the context of SU(5). Let me list a few of them:

1) Assume that the phase transition from SU(5) to SU(3)xSU(2)xU(1) is strongly first order, i.e., it proceeds only after a certain amount of supercooling. The parameters of the SU(5) higgs potential can be adjusted in order to achieve this. Unfortunately, this still probably leads to the production of an unacceptably large number density of monopoles through the Kibble mechanism.

2) Inflationary Scenario: Inflation presumably can overcome the monopole problem. However, as I mentioned earlier on, the scenario cannot be implemented in SU(5)[16] (nor for that matter in any of the known GUTS).

3) One possible way of overcoming the monopole problem in SU(5) is to add extra higgs field to the system and arrange parameters in such a way that the symmetry breaking pattern of SU(5) in the very early universe is very different from what is suggested by the simplest version[17]. For instance, one could envisage the following scenario

$$SU(5) \rightarrow SU(3) \rightarrow SU(3) \times SU(2) \times U(1) \rightarrow SU(3) \times U(1)_{em}$$

This model predicts essentially zero monopoles in the present universe. Although perfectly logical, I do not regard this resolution of the monopole problem as being particularly attractive. However, it can only be excluded by looking for GUT monopoles and finding one!

I shall now argue that the inevitable existence of an intermediate mass scale in axion models can be exploited to resolve the cosmological monopole problem[18]. To be specific consider the SO(10) axion model:

$$SO(10) \xrightarrow[M_x \sim 10^{15} GeV]{} SU(4)_c \times SU(2) \times U(1) \xrightarrow[M_c \sim 3.5 \times 10^{11} GeV]{} SU(3) \times SU(2) \times U(1)_y$$

Monopoles are produced at the phase transition where the SO(10) symmetry breaks down to $SU(4)_c \times SU(2) \times U(1)$. This transition takes place at a critical temperature $T_c \sim 10^{15}$ GeV. We will assume that the initial relative monopole density

$$r_{in} \gtrsim 10^{-10}$$

Subsequent monopole - antimonopole annihilation reduces the relative density, as previously discussed, to

$$r_f \sim 10^{-10}$$

at temperatures of order 10^{12} GeV.

The parameters of the theory can be chosen so that the zero temperature effective potential for the breaking of $SU(4)_c \times SU(2) \times U(1)$ down to $SU(3) \times SU(2) \times U(1)_y$ is of the Coleman-Weinberg type[19]. In this case, as the universe cools below a critical temperature $T_{c1} \sim M_c$, the $SU(4)_c \times SU(2) \times U(1)$ phase becomes metastable. The vacuum energy density of this phase soon dominates over the radiation energy density, and the universe enters an exponentially expanding de Sitter state. Gravitational and thermal effects destabilize the $SU(4)_c \times SU(2) \times U(1)$ phase at a temperature T_{c2} of order the Hawking temperature T_H of this phase

$$T_{c2} \sim T_H = \frac{H}{2\pi} \sim \frac{M_c^2}{M_{PL}} \sim 10^4 \text{ GeV}$$

Here H is the Hubble constant of the de Sitter state and $M_{PL} \simeq 1.2 \times 10^{19}$ GeV is the Planck mass. The transition to the $SU(3) \times SU(2) \times U(1)_y$ phase is rapidly completed at T_{c2} and the latent heat is released. The Universe reheats to a temperature T_R of order $(1-3) \times 10^{11}$ GeV and the relative monopole density is <u>diluted</u> to[18]

$$r(T_R) \sim 10^{-10} \left(\frac{T_{c_2}}{T_R}\right)^3 \sim 10^{-32} - 10^{-31}$$

This value for r is consistent with the cosmological bounds from nucleosynthesis and from the observed values of the Hubble

constant and the deceleration paramter. The cosmological monopole problem is therefore resolved. Moreover, the predicted monopole density is just about enough to sustain an anisotropic galactic monopole flux at the level of the Parker bound for 10^{10} years. Thus, a monopole flux of[18]

$$F \sim (10^{-8} - 10^{-7})\ \mathrm{cm}^{-2}\ \mathrm{yr}^{-1}$$

may still exist in our galaxy.

Note that the predicted monopole flux is compatible with a recent upper bound on it derived from considerations of observational limits on the diffuse ultraviolet and X-ray background[20]. This latter bound happens to coincide with well known bounds on 10^{16} GeV mass monopoles obtained in ref (21).

The resolution of the cosmological monopole problem in the manner described above implies that the observed baryon asymmetry in the universe gets created after completion of the intermediate phase transition. One must therefore require that there exist Higgs bosons with masses of order 10^{11} GeV which have $\Delta B \neq 0$ decay modes. The out of equilibrium decay of these bosons can create the observed baryon asymmetry.

Thus, the SO(10) axion model with an intermediate mass scale of about $10^{11} - 10^{12}$ GeV is able to overcome the problems explicitly listed in the abstract of the talk. Let me briefly discuss two other topics in the context of this model. First consider proton decay. The requirement that there exist higgs bosons with masses of order 10^{11} GeV and with $\Delta B \neq 0$ couplings to ordinary fermions suggests that nucleon decay mediated by these bosons competes with, and perhaps even dominates, the usual gauge boson mediated decays. Thus, processes like $p \to \mu^+ K^0$ may dominate over the usual decay mode of the proton such as $p \to e^+ \pi^0$.

Finally, note that B-L is spontaneously broken in this SO(10) model at the intermediate scale of about 10^{11} GeV. Following ref (22), this implies that the tau neutrino has a mass

$$m_{\nu_\tau} \sim \frac{m_t^2}{M_{B-L}} \gtrsim \text{several (eV), for } m_t \gtrsim 20 \text{ GeV.}$$

We thus have the intriguing possibility that both axions and neutrinos contribute significantly to the energy density of the universe.

Gauge Hierarchy Problem and Supersymmetry

The inability of the standard GUTS to explain small mass

ratios such as $m_W/m_X \sim 10^{-13}$ etc. in a satisfactory manner is referred to as the gauge hierarchy problem. For instance, in SU(5) $m_W/m_X \sim 10^{-13}$ is obtained only if the parameters of the theory are fine tuned to one part in 10^{26}. Moreover, ordinary perturbation theory does not respect fine tuning, so that adjustments to the parameters must be made in each order of the expansion. The last problem perhaps can be overcome if m_W/m_X happens to lie near a fixed point[23].

It was hoped that supersymmetry (SUSY) may overcome the gauge hierarchy problem. However, this is not borne out by recent calculations. Fine tuning remains part and parcel also of SUSY GUTS. This seems to hold both for global and N=1 local SUSY GUTS. Moreover, more often than not, the merger of cosmology and SUSY leads to unacceptable consequences.[24] Although one would like to think that an attractive idea like supersymmetry should play a role in particle physics, this has not yet been satisfactorily realized.

Concluding Remarks

The presence in GUTS of an intermediate mass scale of about 10^{11} – 10^{12} GeV can help resolve a number of apparently unrelated problems. We must be prepared to go beyond (SU(5). One can think of at least two ways of achieving this. Either by going to "standard" larger GUTS such as SO(10), or by attempting to combine SU(5) with a family symmetry. An example incorporating the first possibility is readily constructed and is described in the text. An elegant example utilizing the second possibility remains to be found. Finally, lcoal supersymmetric GUTS also require an intermediate scale of about 10^{11} GeV[25]. Is there any connection between these two intermediate scales?

Acknowledgement

I would like to thank Abdus Salam for arousing my interest in the subject and Harvey Newman for arranging an extremely nice conference in Erice.

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